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SCATTERING OF RADIATION IN AN ANISOTROPIC TURBULENT MEDIUM

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Scattering of laser radiation on density fluctuations in propagation of radiation through an anisotropic turbulent medium is analyzed. It is shown that the deviation angles in turbulent gas flows at atmospheric pressure equal $\sim 10^{-5}-10^{-4}$ *rad and can be detected by means of speckle photography. A statistical analysis of two-dimensional fields of deviation angles makes it possible to evaluate three-dimensional density correlation functions in a turbulent flow. It is shown that taking account of the turbulence anisotropy leads to distributions of the laser-radiation intensity over deviation angles that deviate substantially from the Gaussian distribution.*

Laser-radiation transport through a turbulent medium and its scattering have repeatedly been the subject of analysis [1-4]. Generally, scattering is described by the Maxwell equations for electromagnetic waves. Propagation and scattering of short waves can be described successfully by methods of geometrical optics based on use of the ray equation.

The approximation of geometrical optics is traditionally used to solve problems of aerothermooptics where the flows are laminar [5]. Calculations of laser-radiation transport through a turbulent medium can also be carried out within the framework of the geometrical-optics approximation [1]. In this case, it should be postulated that the dimensions of the inhomogeneities in the medium are large compared to the radiation wavelength, i.e., $l_t >> \lambda$, where l_t is the characteristic turbulence microscale determined, e.g., from the correlation function of density pulsations and λ is the radiation wavelength. As has been shown by a recent analysis [6], the restrictions on the applicability of geometrical optics also apply to the dimensions of the turbulent medium L through which the radiation propagates.

This condition can be written as follows:

$$L << l_t^2/\lambda . \tag{1}$$

It is evident from relationship (1) that for the characteristic size $L \sim 0.1$ m (laboratory modeling of turbulence) the minimum sizes of turbulence microscales whose effect on radiation propagation is described within the framework of geometrical optics are ≤ 1 mm. This value also corresponds well to both the spatial resolution of speckle photography and modern possibilites of computer simulation of turbulence. Thus, three-dimensional fields of gas-dynamic parameters were calculated by Gerz et al. [7] on grids with dimensions up to $160 \times 160 \times 160$ using the direct numerical simulation (DNS) method. Such a high spatial resolution achieved in these calculations made it possible to describe small-scale turbulent vortices with Reynolds numbers based on l_t of the order of 100. However, to accomplish this, Gerz et al. had to use a computer with the presently maximum possible RAM size.

We calculated propagation of radiation through a turbulent medium on a $64 \times 64 \times 64$ grid using turbulence-field data obtained by the DNS method [7] that were kindly provided by Dr. Gerz.

The equation of radiation propagation within the geometrical-optics approximation (the eikonal equation) in differential form is as follows:

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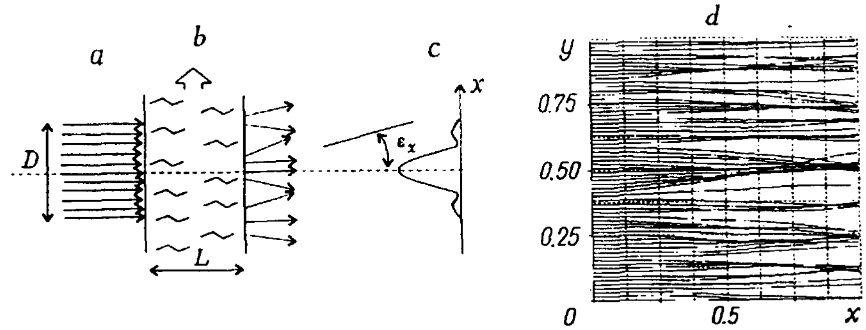

Fig. 1. Geometry of the problem: a) collimated laser radiation; b) turbulent flow; c) intensity distribution in the far-field zone; d) trajectories of light rays in the turbulent-flow region.

$$\frac{d}{ds}\left[n\,(\mathbf{r})\,\frac{d\mathbf{r}}{ds}\right] = \nabla n\,(\mathbf{r})\,, \tag{2}$$

where $\mathbf{r}(s)$ is the radius vector of a point located on the light ray. Here $\mathbf{r} = x\mathbf{i} + y\mathbf{j} + z\mathbf{k}$ and ds is the elementary arc length along the ray. We introduce the following notation:

$$t = \int \frac{ds}{n}\,, \quad dt = \frac{ds}{n}\,, \quad \mathbf{T} = \frac{d\mathbf{r}}{dt}\,. \tag{3}$$

Equation (2) can then be written as follows:

$$\frac{d^2\mathbf{r}}{dt^2} = n\nabla n\,(\mathbf{r}) \quad \text{or} \quad \frac{d^2\mathbf{r}}{dt^2} = \frac{1}{2}\nabla n^2\,(\mathbf{r})\,. \tag{4}$$

The coordinates of the vector T are direction cosines:

$$\mathbf{T} \equiv \frac{d\mathbf{r}}{dt} \equiv n\,\frac{d\mathbf{r}}{ds} \equiv n\,\frac{dx}{ds}\,\mathbf{i} + n\,\frac{dy}{ds}\,\mathbf{j} + n\,\frac{dz}{ds}\,\mathbf{k} \equiv n\cos\alpha\mathbf{i} + n\cos\beta\mathbf{j} + n\cos\gamma\,\mathbf{k}\,,$$

where α, β, and γ are the angles between the ray direction and the x-, y-, and z-axes, respectively.

The ray trajectory was calculated by the Runge–Kutta method. The current coordinates and inclination angles of the ray were calculated for small increments $\Delta t = l/n\Delta s$ [8]:

$$R_{n+1} = R_n + \Delta t\left[T_n + \frac{1}{6}(A + 2B)\right], \quad T_{n+1} = T_n + \frac{1}{6}(A + 4B - C)\,, \tag{5}$$

where

$$A = \Delta t D\,(R_n)\,; \tag{6}$$

$$B = \Delta t D\left(R_n + \frac{\Delta t}{2}\,T_n + \frac{1}{8}\,\Delta t A\right); \tag{7}$$

$$C = \Delta t D\left(R_n + \Delta t T_n + \frac{1}{2}\,\Delta t B\right); \tag{8}$$

$$R = \begin{pmatrix} x \\ y \\ z \end{pmatrix}; \quad T = \begin{pmatrix} T_x \\ T_y \\ T_z \end{pmatrix} \equiv n \begin{pmatrix} dx/ds \\ dy/ds \\ dz/ds \end{pmatrix}; \tag{9}$$

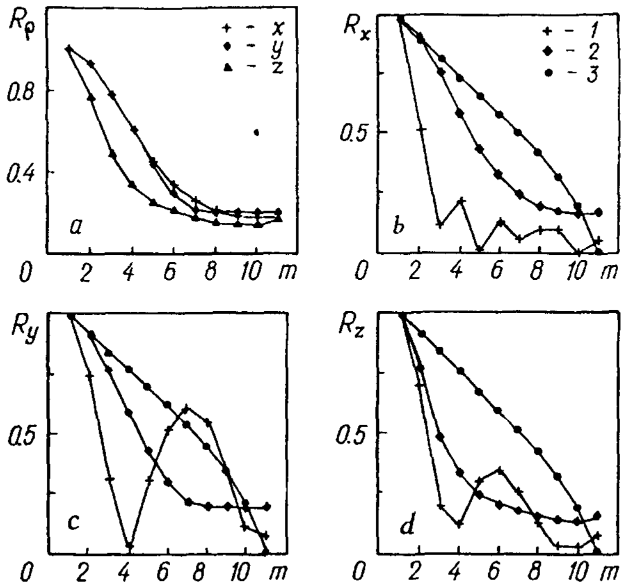

Fig. 2. Two-point density correlation functions for a turbulent flow along different coordinate axes (a) (m is the cell number along the corresponding axis), correlation functions of the density and deviation angles of light rays in a turbulent flow along the x-, y-, and z-axes (b, c, and d, respectively); 1) deviation-angle function calculated by direct numerical simulation; 2) density function calculated from Dr. Gerz's data for the corresponding coordinate; 3) density function recovered from deviation angles using the Ehrbeck–Merzkirch integral).

$$D \equiv n \begin{pmatrix} \partial n/\partial x \\ \partial n/\partial y \\ \partial n/\partial z \end{pmatrix} \equiv \frac{1}{2} \begin{pmatrix} \partial (n^2)/\partial x \\ \partial (n^2)/\partial y \\ \partial (n^2)/\partial z \end{pmatrix} . \tag{10}$$

Using the above notation, we can write the trajectory equation in matrix form:

$$\frac{d^2R}{dt^2} = D(R) . \tag{11}$$

Figure 1 presents the geometry of the problem and results of direct numerical simulation for a light wave propagating through different cross sections of a turbulent flow. The turbulent-field variations have been intentionally enhanced by a factor of 100 to visualize the optical rays. It is evident from results of calculations that the deflection angles ε of the optical rays in a turbulent medium with $L \sim 0.1$ m and a level of temperature fluctuations $\sqrt{\langle T^2 \rangle} \sim 5-10$ K equal $10^{-4}-10^{-5}$ rad, which corresponds quite well to the sensitivity range of both double- and single-exposure speckle photography. A statistical analysis of the deflection-angle field shows that $\langle \varepsilon \rangle \to 0$ when the number of cells in the grid exceeds 30 × 30. For the 64 × 64 grid used in the calculation of the two-dimensional field of light-ray deflection angles, the "residual" values of the average fluctuating quantities do not exceed 1%.

Figure 2 presents results of calculations of correlation functions for the original density field in a three-dimensional turbulent flow and deflection angles in the resulting two-dimensional map. A comparison of these functions indicates that correction of experimental data by an integral transform should be used to recover the three-dimensional correlation function from data on the two-dimensional correlation function obtained from speckle photography, as has been described earlier [9]. The same figure shows density correlation functions calculated from light-ray deviation angles using such an integral transform. It is evident that they tend to approach the original

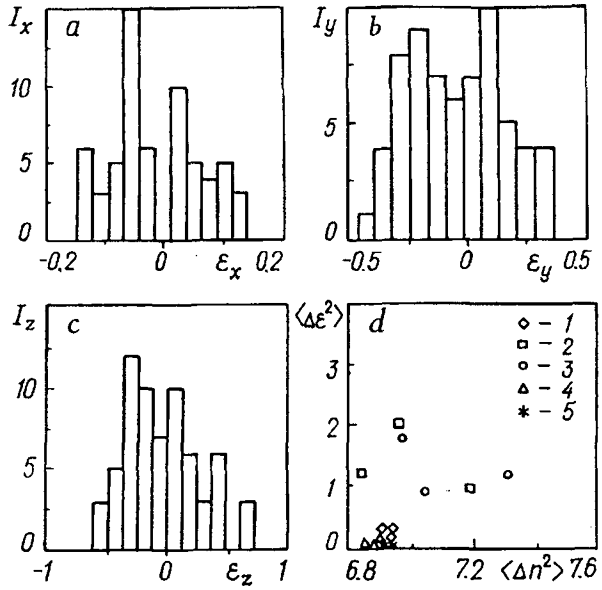

Fig. 3. Distribution of radiation intensity (arbitrary units) over deviation angles (rad) in propagation of collimated laser radiation through an anisotropic turbulent medium (deviation angles along the x-, y-, and z-axes (a, b, and c) were calculated for corresponding central cross sections of a cube $(y - z)$, $(x - z)$, and $(x - y)$ for 64 probing lines), and comparison of root-mean-square deviations of light rays with root-mean-square pulsations of the refractive index for all 15 variants of the calculations (d).

density correlation functions obtained from turbulence simulation by the DNS method. Especially good agreement is evident for small-scale turbulence for a vortex size close to l_t. Disagreement of the correlation functions is observed for vortex sizes exceeding 10–15 l_t.

Thus, results of direct numerical simulation of propagation of laser radiation in a turbulent medium demonstrate the possibility in principle of evaluation of three-dimensional correlation functions of a turbulence field from experimental data obtained by the speckle-photography technique. To provide a statistical analysis of the experimental quantities described, automated transfer of them to a computer with a density corresponding to grids with dimensions of no less than 64 × 64 is required.

The presence of pulsations leads to additional radiation scatter that can be characterized by the root-mean-square deviation angle. As has been shown in [2], this quantity can be calculated as an integral of the gradient of the density correlation function $R_\rho(r)$:

$$\langle\Delta\varepsilon^2\rangle = -\langle\Delta n^2\rangle LK \int_0^\infty \frac{1}{r^2}\frac{d}{dr}\left(\frac{r^2\, dR_\rho(r)}{dr}\right) dr\,, \tag{12}$$

where K is the Gladstone–Dale constant, and n is the refractive index,

$$(n - 1) = K\rho\,. \tag{13}$$

For the particular case of a density correlation function approximated by the Gaussian function

$$R_\rho(r) = \exp(-r^2/l_t^2)\,, \tag{14}$$

relationship (12) can be solved analytically:

$$\langle \Delta\varepsilon^2 \rangle = \pi \langle \Delta n^2 \rangle \frac{L}{L_t}, \tag{15}$$

where L_t is the turbulence macroscale.

Thus, for isotropic turbulence, the root-mean-square deviation of the scattering angle is determined by the root-mean-square density pulsations in the turbulent flow. Relationship (15) is basic for measurements of the intensity of turbulent pulsations by single-exposure speckle photography with an extended exposure time [10]. We verify the validity of relationship (15) by direct numerical simulation of refraction in a turbulent medium based on the geometrical-optics approximation.

Flow anisotropy was determined from results of calculations of density correlation functions along different coordinate axes. Figure 3 presents results of calculations of light-ray propagation for a flow with maximum anisotropy. This variant simulates shear flow with a refractive-index gradient along the z-axis. Trajectories of light rays propagating through a turbulent flow are presented in [11]. For clarity, the light-ray deviation angles ε are multiplied by a factor of 10^4 in Fig. 3a, b, and c. Here the root-mean-square pulsations of the refractive index along the x-, y-, and z-axes were equal to $\langle \Delta n^2 \rangle_{yz} = 6.93 \cdot 10^{-8}$, $\langle \Delta n^2 \rangle_{xz} = 0.690 \cdot 10^{-7}$, and $\langle \Delta n^2 \rangle_{xy} = 0.693 \cdot 10^{-7}$, respectively, and the corresponding root-mean-square deviations of the light rays were equal to $\langle \Delta\varepsilon^2 \rangle_x = 0.57 \cdot 10^{-10}$, $\langle \Delta\varepsilon^2 \rangle_y = 0.402 \cdot 10^{-9}$, and $\langle \Delta\varepsilon^2 \rangle_z = 0.938 \cdot 10^{-9}$, respectively.

As is shown by histograms, the distribution of light rays over deviation angles has an irregular character that is individual for each particular realization of the cross section of the refractive-index field in the turbulent flow. It should be noted that the (64 × 64-cell) grid used in calculations of deviation angles corresponds well to the spatial resolution of speckle photography [11]. Modern automated specklogram-processing systems based on PC-coupled CCD-arrays make it possible to recover the vector field of deviation angles of light rays on grids with dimensions of (50 × 50) ... (100 × 100) and higher, thus providing recovery of three-dimensional correlation functions (of, e.g., density) from experimentally determined two-dimensional correlation functions of light-ray deviation angles [12].

Figure 3d presents final results of calculations for 15 cross sections (three cross sections for each of five variants) in the form of the dependence $\langle \Delta\varepsilon^2 \rangle$ ($\langle \Delta n^2 \rangle$) (for clarity, all values are multiplied by a factor of 10^8). These data show that relationship (15) holds only approxinately and the deviation of results of calculations from the theoretical dependence can be substantial for an anisotropic flow. The laser-radiation divergence in the far-field zone for a single passage of radiation through an active laser medium will be determined by a two-dimensional Fourier transform of the radiation amplitude at the resonator exit. In the case of absence of turbulent pulsations, the divergence can be determined only by the diffraction on the aperture D and can be expressed in terms of the squared Bessel function of the first order:

$$I_{\mathrm{dif}}(\varepsilon) = I_0 J_1^2 \left(\frac{\pi \varepsilon D}{\lambda} \right). \tag{16}$$

For isotropic turbulence, we describe the effect of refractive -index pulsations on radiation divergence in terms of the turbulence microscale l_t [2]:

$$I_t(\varepsilon) = I_0 \exp \left(- \frac{\pi \varepsilon^2 l_t^2}{4\lambda^2} \right). \tag{17}$$

As is evident, in this case radiation divergence is described by a Gaussian function. In cases where the effect of turbulence is small, the intensity distribution in the far-field zone can be obtained as a convolution of distributions (16) and (17). For intense turbulent pulsations, the diffraction component (16) can be neglected and radiation divergence can be described by relationship (17). At the same time, as is shown by results of direct calculations presented in Fig. 3, the intensity distribution for anisotropic flows can deviate substantially from a Gaussian distribution. Numerous additional maxima are distinguished in the angular distributions obtained, which increase the divergence of the laser radiation. Thus, taking account of the turbulence anisotropy is sufficient for determination of the actual picture of radiation divergence in flow laser systems with a high level of turbulent pulsations.

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